Section I 10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

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1. Which of the following is the angle between the vectors $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

A. 10°

B. 63°

C. 140°

D. 117°

2. Which of the following equal to $\cos\theta$

A. $\sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}$

B. $2\cos^2\frac{\theta}{2} + 1$

C. $1 - 2\cos^2\theta$

D. $\dfrac{\sin\theta}{\tan\theta}$

3. The polynomial $P(x) = x^3 - x^2 - 5x - 3$ has a double root at $x = \alpha$. What is the value of α?

A. $\dfrac{-5}{3}$

B. -1

C. 1

D. $\dfrac{5}{3}$

4. Which of the following is equivalent to $\int \sin x \cos x \, dx$?

A. $-\cos 2x + c$

B. $-\frac{1}{2}\cos 2x + c$

C. $-\frac{1}{4}\cos 2x + c$

D. $\frac{1}{4}\cos 2x + c$

5. What is the domain of $y = 3\cos^{-1}(2x + 1)$?

A. Domain $[-1, 0]$

B. Domain $(1, 0)$

C. Domain $[-\frac{1}{2}, 0]$

D. Domain $[\frac{1}{2}, 0]$

6. In a Mathematics class a teacher can award one of 4 grades, A, B, C or D to each student. What is the minimum number of students required so that at least 8 students are guaranteed to receive the same grade?

A. 28

B. 29

C. 32

D. 33

7. Which of the following is the range of the function $\dfrac{1}{x^2+1}$

A. $(-\infty, \infty)$

B. $(-\infty, 1]$

C. $(0, 1]$

D. $[0, 1]$

8. A curve has an asymptote at $x = \dfrac{\pi}{3}$. Which of the following could be the equation of the curve?

A. $y = \sec\left(x - \dfrac{\pi}{3}\right)$

B. $y = \sec\left(x + \dfrac{\pi}{3}\right)$

C. $y = \cot\left(x - \dfrac{\pi}{3}\right)$

D. $y = \operatorname{cosec}\left(x + \dfrac{\pi}{3}\right)$

9. Which of the following is the primitive of $\dfrac{4}{\sqrt{9-x^2}}$?

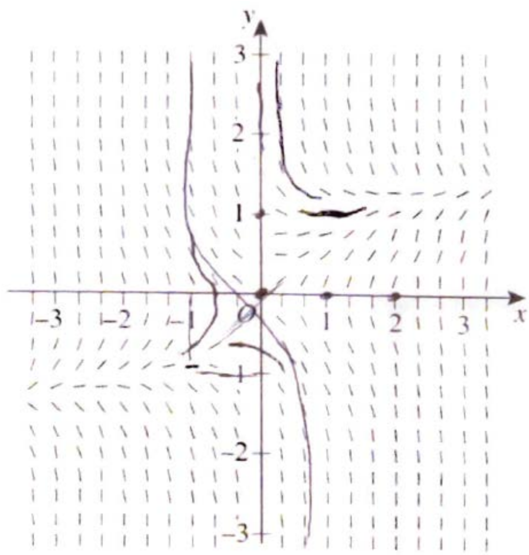
A. $\dfrac{4}{3}\sin^{-1}\dfrac{x}{3} + c$

B. $\dfrac{4}{3}\sin^{-1} 3x + c$

C. $4\sin^{-1} 3x + c$

D. $4\sin^{-1}\dfrac{x}{3} + c$

10. Which of the following best represents the differential equation shown in the slope field?



A. $\dfrac{dy}{dx} = \dfrac{x}{y} - y^2$

B. $\dfrac{dy}{dx} = \dfrac{x}{y} + y^2$

C. $\dfrac{dy}{dx} = -\dfrac{x}{y} - y^2$

D. $\dfrac{dy}{dx} = -\dfrac{x}{y} + y^2$

Section II 60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available
For questions in Section II, your response should include relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Solve $\dfrac{3}{2x-5} \leq -1$ **3**

(b) Find the derivative of $\cos^{-1}\left(\dfrac{3x}{2}\right)$ **1**

(c) For what values of m are the distinct vectors $\begin{pmatrix} m \\ 2m+6 \end{pmatrix}$ and $\begin{pmatrix} m+1 \\ -1 \end{pmatrix}$ perpendicular? **2**

(d) Find $\displaystyle\int \cos^2 4x \, dx$ **2**

(e) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \cos x \sin^3 x \, dx$ **3**

(f) For what values of m is the polynomial $x^2-(m+2)x+2(m+2)$positive for all x **2**

(g) Consider the expansion $(2x-p)^9$. The coefficient of x^6 is -344064. **2**
Find the value of p

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Find $\displaystyle\int \frac{dx}{25+4x^2}$ **2**

(b) Given that $\sin\theta = \dfrac{3}{4}$ and $\dfrac{\pi}{2} \leq \theta \leq \pi$ determine the exact value of $\tan 2\theta$ **3**

(c) (i) Find the derivative of $x \log_e x - x$ **1**

(ii) Hence evaluate $\displaystyle\int_{\sqrt{e}}^{e} \log_e x \, dx$ **2**

(d) A mixed volleyball team of eight players is selected from ten males and nine females **2**
In how many ways can this be done if the team must have at least 2 female players

(e) The vectors $\vec{u} = \begin{pmatrix} a \\ 3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ are parallel. Find the value of a **2**

(f) The polynomial $P(x) = (x-p)^3 + q$ is zero at $x=1$ and when divided by x **3**
the remainder is -7. Find the possible values of p

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\int_{-1}^{3} x\sqrt{3-x}\,dx$ using the substitution $u = 3 - x$. **4**

(b) (i) Express $\sqrt{3}\cos x + \sin x$ in the form $R\cos(x-\alpha)$ where $R > 0$ and α is acute. **2**

(ii) Hence solve $\sqrt{3}\cos x + \sin x = 1$ for $0 \le x \le 2\pi$. **2**

(c) Use the process of Mathematical induction to prove **3**

$2\times 1! + 5\times 2! + 10\times 3! + \ldots + (n^2+1)n! = n(n+1)!$ for all positive integers n.

(d) A population of mice in a meadow after t years satisfies the logistic **4**

differential equation $\frac{dP}{dt} = \frac{3P}{2500}(2500-P)$, where the initial population of

mice is 500.

Given $\frac{1}{P} + \frac{1}{2500-P} = \frac{2500}{P(2500-P)}$, solve the differential equation to find the

population P of the mice at time t. Express your answer in the form

$P = \frac{A}{1-Be^{-kt}}$ where A, B and k are integers.

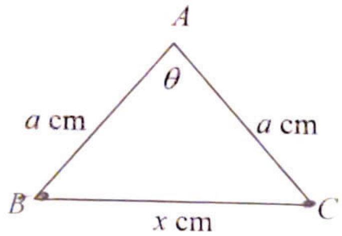
End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Solve the differential equation $(x^2+1)\dfrac{dy}{dx} = 6xy$ where $x = 1$ and $y = 2$ giving your answer as y in terms of x. **3**

(b) Given that $y = e^{2x} + e^{-2x}$, determine the values of constants a and b that satisfy the following differential equation $\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + by = 5e^{2x} + e^{-2x}$. **4**

(c) In the triangle ABC, $AB = AC = a$ cm. The angle BAC is increasing at the rate of 2 radians/min. Let $\angle BAC = \theta$ radians and $BC = x$ cm.



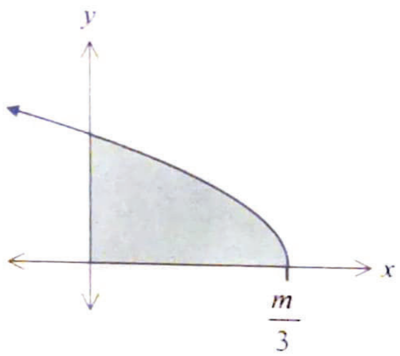
NOT TO SCALE

(i) Show that $x = a\sqrt{2-2\cos\theta}$. $x = a\sqrt{2-2\cos\theta}$ **1**

(ii) Show that $\dfrac{dx}{d\theta} = \dfrac{a\sin\theta}{\sqrt{2-2\cos\theta}}$. **1**

(iii) Determine, in terms of a, the rate with respect to time at which BC is increasing when $\theta = \dfrac{\pi}{3}$ radians. **2**

(d) Let $f(x) = \sqrt{m-3x}$ for $x < \dfrac{m}{3}$. The graph of $y = f(x)$ is shown. **4**



The area enclosed by the graph $y = f(x)$, the x-axis and the y-axis is rotated about the y axis. Find the value of m such that the volume of the solid formed is $\dfrac{5000\pi}{27}$ cubic units.

End of Paper

MATHEMATICS EXTENSION 1 TRIAL 2022

1. $\underset{\sim}{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

$\cos\theta = \dfrac{a \cdot b}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

$= \dfrac{(3 \times -1 + 4 \times 2)}{\sqrt{9+16}\sqrt{1+4}}$

$= \dfrac{5}{5\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

$\theta = \cos^{-1}\dfrac{1}{\sqrt{5}}$

$\fallingdotseq 63^\circ$ (B)

2. (D) $\dfrac{\sin\theta}{\tan\theta} = \dfrac{\sin\theta}{\frac{\sin\theta}{\cos\theta}}$

$= \sin\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= \cos\theta$

3. $P(x) = x^3 - x^2 - 5x - 3$

$P'(x) = 3x^2 - 2x - 5$

Double root $\therefore P'(a) = 0$

$0 = 3a^2 - 2a - 5$ $\quad \left[\begin{array}{l} P\ -15 \\ S\ -2 \\ F\ -5, 3 \end{array}\right.$

$0 = 3a^2 + 3a - 5a - 5$

$0 = 3a(a+1) - 5(a+1)$

$0 = (a+1)(3a-5$

$\therefore a = -1$ or $\dfrac{5}{3}$

$P(-1) = (-1)^3 - (-1)^2 - 5(-1) - 3$

$= -1 - 1 + 5 - 3$

$= 0$

$\therefore P(-1) = 0 \quad P'(-1) = 0$

$\therefore$ (B)

4. $\displaystyle\int \sin x \cos x \, dx = \frac{1}{2}\int \sin 2x \, dx$

$= \dfrac{1}{2}\left(-\dfrac{\cos 2x}{2}\right) + c$

$= -\dfrac{\cos 2x}{4} + c$

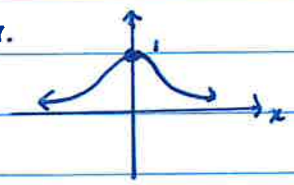
(C)

5. $-1 \leq 2x + 1 \leq 1$

$-2 \leq 2x \leq 0$

$-1 \leq x \leq 0$ $\quad [-1, 0]$ $\quad$ (A)

6. $4 \times 7 + 1 = 29$ $\quad$ (B)

7. 

$0 < y \leq 1$

$(0, 1]$ $\quad$ (C)

8. A. $y = \sec 0 = \dfrac{1}{\cos 0} = 1$

B. $y = \sec\left(\dfrac{2\pi}{3}\right)$

C. $y = \cot(0)$ $\quad \dfrac{\cos 0}{\sin 0}$ $\quad \sin 0 = 0 \ \therefore$ (C)

undefined at $x = 0$

9. $\displaystyle\int \frac{4}{\sqrt{3^2 - x^2}}\,dx = 4\sin^{-1}\frac{x}{3} + c$ $\quad$ (D)

(A or D)

10. (A) $x = 1$ $y = 1$ $\dfrac{dy}{dx} = 1 - 1 = 0$ ✓

$x = -2$ $y = -1$ $\dfrac{dy}{dx} = 1$ ✓

D $\dfrac{dy}{dx} = -2 + 1$ ✗

$= -1$

$\therefore$ (A)

1	B
2	D
3	B
4	C
5	A
6	B
7	C
8	C
9	D
10	A.

(15 Marks)

11 a) $\dfrac{3}{2x-5} \leq -1$

critical values

$x \neq \dfrac{5}{2}$

$\dfrac{3}{2x-5} = -1$

$3 = -2x + 5$

$2x = 2$

$x = 1$

test $x = 2$ $\quad \dfrac{3}{-1} \leq -1$ ✓

$1 \leq x < 2\frac{1}{2}$ $\quad$ [3]

b) $y = \cos^{-1} \dfrac{3x}{2}$

$\dfrac{dy}{dx} = \dfrac{-3}{\sqrt{4-9x^2}}$

$a = 2$
$f(x) = 3x$
$f'(x) = 3$

[1]

c) perpendicular when

$\begin{pmatrix} m \\ 2m+6 \end{pmatrix} \cdot \begin{pmatrix} m+1 \\ -1 \end{pmatrix} = 0$

$m(m+1) + -1(2m+6) = 0$

$m^2 + m - 2m - 6 = 0$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$\therefore m = 3$ or -2 $\quad$ [2]

d) $\displaystyle\int \cos^2 4x \, dx = \int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 8x\right) dx$

$= \frac{1}{2}x + \frac{1}{2} \frac{\sin 8x}{8} + c$

$= \frac{1}{2}x + \frac{1}{16} \sin 8x + c$ $\quad$ [2]

e) $\displaystyle\int_0^{\frac{\pi}{6}} \cos x \sin^3 x \, dx$

$= \left[\dfrac{\sin^4 x}{4}\right]_0^{\frac{\pi}{6}}$

$= \dfrac{(\sin \frac{\pi}{6})^4}{4} - \dfrac{\sin^4 0}{4}$

$= \left(\frac{1}{4} \times \frac{1}{2^4}\right) - 0$

$= \dfrac{1}{64}$ $\quad$ [3]

f) For positive $\Delta < 0$

$\Delta = b^2 - 4ac$

$= [-(m+2)]^2 - 4(2(m+2))$

$= m^2 + 4m + 4 - 8m - 16$

$= m^2 - 4m - 12$

$= (m-6)(m+2)$ $\quad m = -2, 6$

$(m-6)(m+2) < 0$ $\quad$ test $m = 0$

$-6 \times 2 < 0$ ✓

$-2 < m < 6$ $\quad$ [2]

g) Coefficient of $x^6 = {}^9C_6 (2x)^6 (-p)^3$

$-344\,064 = -5376 p^3$

$64 = p^3$

$p = 4$ $\quad$ [2]

(15 Marks)

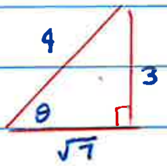
12 a) $\displaystyle\int \frac{dx}{25+4x^2} = \frac{1}{2}\int \frac{2\,dx}{5^2+(2x)^2}$

$a=5$
$f(x)=2x$
$f'(x)=2$

$= \frac{1}{5} \times \frac{1}{2} \tan^{-1} \frac{2x}{5} + c$

$= \frac{1}{10} \tan^{-1} \frac{2x}{5} + c$ [2]

b) $\sin\theta = \frac{3}{4}$ (Q2)



$\tan\theta = -\frac{3}{\sqrt{7}}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2 \times -\frac{3}{\sqrt{7}}}{1-\left(-\frac{3}{\sqrt{7}}\right)^2}$

$= \frac{-\frac{6}{\sqrt{7}}}{1-\frac{9}{7}}$

$= -\frac{6}{\sqrt{7}} \times -\frac{7}{2}$

$= \frac{21}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{21\sqrt{7}}{7}$

$= 3\sqrt{7}$ [3]

c)

(i) $y = x\log_e x - x$

$u = x$
$u' = 1$
$v = \log_e x$
$v' = \frac{1}{x}$

$\frac{dy}{dx} = x \times \frac{1}{x} + \log_e x \times 1 - 1$

$= 1 + \log_e x - 1$

$= \log_e x$ [1]

(ii) $\displaystyle\int_{\sqrt{e}}^{e} \log_e x \, dx = \left[x\log_e x - x\right]_{\sqrt{e}}^{e}$

$= (e\log_e e - e) - (\sqrt{e}\log_e \sqrt{e} - \sqrt{e})$

$= (e-e) - (\sqrt{e}\log_e e^{\frac{1}{2}} - \sqrt{e})$

$= 0 - (\frac{1}{2}\sqrt{e}\log_e e - \sqrt{e})$

$= 0 - \frac{1}{2}\sqrt{e} + \sqrt{e}$

$= \frac{1}{2}\sqrt{e}$ [2]

d) 8 players 10 males 9 females

At least 2 female = Total − (0F + 1F)

$= {}^{19}C_8 - \left[{}^{9}C_0 \times {}^{10}C_8 + {}^{9}C_1 \times {}^{10}C_7\right]$

$= 74\,457$ [2]

(15 Marks)

12(e) $\underset{\sim}{u} = \begin{pmatrix} a \\ 3 \end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

For parallel $\begin{pmatrix} a \\ 3 \end{pmatrix} = k\begin{pmatrix} 2 \\ 7 \end{pmatrix}$

$3 = k \times 7$

$\frac{3}{7} = k$

$\therefore\ a = \frac{3}{7} \times 2$

$\underline{a = \frac{6}{7}}$ [2]

(f) $P(x) = (x-p)^3 + q$

$\left.\begin{array}{l} P(1) = 0 \\ P(0) = -7 \end{array}\right\}$

$P(1) = (1-p)^3 + q$

$0 = (1-p)^3 + q$ ①

$P(0) = (-p)^3 + q$

$-7 = (-p)^3 + q$

$-7 = -p^3 + q$

$-7 + p^3 = q$ ②

Sub ② into ①

$0 = (1-p)^3 - 7 + p^3$

$0 = 1 - 3p + 3p^2 \cancel{-p^3} - 7 \cancel{+p^3}$

$0 = 3p^2 - 3p - 6$

$0 = p^2 - p - 2$

$0 = (p-2)(p+1)$

$\underline{\therefore p = 2 \text{ or } -1}$ [3]

13) $\int_{-1}^{3} x\sqrt{3-x}\,dx$

$\left[\begin{array}{l} u = 3 - x \\ \frac{du}{dx} = -1 \\ -du = dx \\ x = 3 \quad u = 0 \\ x = -1 \quad u = 4 \\ x = 3 - u \end{array}\right.$

$= -\int_{4}^{0} (3-u)\,u^{\frac{1}{2}}\,du$

$= \int_{0}^{4} (3u^{\frac{1}{2}} - u^{3/2})\,du$

$= \left[\frac{3u^{3/2}}{\frac{3}{2}} - \frac{u^{5/2}}{\frac{5}{2}}\right]_0^4$

$= \left[2u^{3/2} - \frac{2}{5}u^{\frac{5}{2}}\right]_0^4$

$= \left[2(4)^{3/2} - \frac{2}{5}(4)^{5/2}\right] - \left[0 - 0\right]$

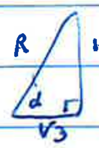
$= 16 - \frac{64}{5}$

$= \underline{\frac{16}{5}}$ [4]

b)(i) $\sqrt{3}\cos x + \sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$

$R\cos\alpha = \sqrt{3}$ $R\sin\alpha = 1$

$\cos\alpha = \frac{\sqrt{3}}{R}$ $\sin\alpha = \frac{1}{R}$



$R = \sqrt{3+1}$

$= 2$

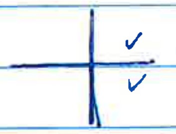
$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\therefore\ \underline{\sqrt{3}\cos x + \sin x = 2\cos(x - \frac{\pi}{6})}$ [2]

(ii) $2\cos(x - \frac{\pi}{6}) = 1$

$\cos(x - \frac{\pi}{6}) = \frac{1}{2}$



rel'd L $= \frac{\pi}{3}$

$\therefore\ x - \frac{\pi}{6} = \frac{\pi}{3}$ or $x - \frac{\pi}{6} = \frac{5\pi}{3}$

$x = \frac{\pi}{3} + \frac{\pi}{6}$ $x = \frac{5\pi}{3} + \frac{\pi}{6}$

$= \underline{\frac{\pi}{2}}$ or $= \underline{\frac{11\pi}{6}}$ [2]

(c) Step 1 : Prove true for $n=1$

$LHS = 2 \times 1!$ $= 2$ $RHS = 1(1+1)!$ $= 2!$ $= 2$ $\therefore$ LHS = RHS

True for $n=1$

Step 2: Assume true for $n=k$

$$2\times 1! + 5\times 2! + 10\times 3! + \cdots + (k^2+1)k! = k(k+1)! \quad *$$

Step 3: Prove true for $n=k+1$

RTP: $\underbrace{2\times 1! + 5\times 2! + 10\times 3! + \cdots + (k^2+1)k!}_{S_k} + [(k+1)^2+1](k+1)! = (k+1)(k+2)!$

$LHS = k(k+1)! + [k^2+2k+1+1](k+1)!$ (by assumption *)

$= (k+1)![k + k^2+2k+2]$

$= (k+1)!\,[k^2+3k+2]$

$= \underbrace{(k+1)!\,(k+2)}(k+1)$

$= (k+2)!\,(k+1)$

$= RHS$

Step 4: Since true for $n=1$, then true for $n=2$, $n=3$ and so on for all positive integers.

[3]

(d) $\frac{dP}{dt} = \frac{3P}{2500}(2500-P)$ Trivial solns: $P=0$, $P=2500$

$$\frac{dP}{\frac{P}{2500}(2500-P)} = 3\,dt$$

$$\int \frac{2500}{P(2500-P)}\,dP = \int 3\,dt$$

$$\int \left(\frac{1}{P} + \frac{1}{2500-P}\right) dP = 3\int dt$$

$$\ln|P| - \ln|2500-P| = 3t + c$$

$$\ln\left|\frac{P}{2500-P}\right| = 3t + c$$

$$\left|\frac{P}{2500-P}\right| = e^{3t+c}$$

$$= e^{3t} \times e^{c}$$

$$= Ae^{3t} \quad \text{where } A = e^{c}$$

$$\frac{P}{2500-P} = Ae^{3t} \quad A = \pm e^{c}$$

when $t=0$, $P=500$: $\frac{500}{2000} = Ae^{0}$

$$\tfrac{1}{4} = A$$

$$\therefore \frac{P}{2500-P} = \frac{1}{4}e^{3t}$$

$$4P = e^{3t}(2500-P)$$

$$4P = 2500e^{3t} - Pe^{3t}$$

$$4P + Pe^{3t} = 2500e^{3t}$$

$$P(4+e^{3t}) = 2500e^{3t}$$

$$P = \frac{2500e^{3t}}{4+e^{3t}} \quad \div e^{3t}$$

$$= \frac{2500}{\frac{4}{e^{3t}} + 1}$$

$$= \frac{2500}{1+4e^{-3t}} \quad B=-4 \quad [4]$$

or $P = \frac{2500}{1-(-4)e^{-3t}}$

(15 Marks)

14 $(x^2+1)\frac{dy}{dx} = 6xy$

a) $\int \frac{dy}{y} = \int \frac{6x}{x^2+1}\,dx$

$\ln|y| = 3\int \frac{2x}{x^2+1}\,dx$

$\ln|y| = 3\ln|x^2+1| + c$

$x=1 \quad y=2 \qquad \ln 2 = 3\ln 2 + c$

$-2\ln 2 = c$

$\therefore \ln y = 3\ln(x^2+1) - 2\ln 2$

$\ln y = \ln(x^2+1)^3 - \ln 2^2$

$y = \frac{(x^2+1)^3}{4}$

[3]

b) $y = e^{2x} + e^{-2x}$

$\frac{dy}{dx} = 2e^{2x} - 2e^{-2x}$

$\frac{d^2y}{dx^2} = 4e^{2x} + 4e^{-2x}$

$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 4e^{2x} + 4e^{-2x} + 2ae^{2x} - 2ae^{-2x} + be^{2x} + be^{-2x}$

$= (4+2a+b)e^{2x} + (4-2a+b)e^{-2x}$

Equating coefficients: $4+2a+b = 5 \qquad 4-2a+b = 1$

$2a+b = 1 \quad (1) \qquad -2a+b = -3 \quad (2)$

$+ \quad -2a+b = -3 \quad (2) \qquad -2a-1 = -3$

$2b = -2 \qquad -2a = -2$

$b = -1 \qquad a = 1$

$\therefore a = 1 \quad b = -1$

[4]

(c) cosine rule:

$c^2 = a^2 + b^2 - 2ab\cos C$

(i) $x^2 = a^2 + a^2 - 2a^2\cos\theta$

$= 2a^2 - 2a^2\cos\theta$ (factorise a^2)

$x = a\sqrt{2-2\cos\theta}$ which is required [1]

ii. $\frac{dx}{d\theta} = a \times \frac{1}{\cancel{2}}(2-2\cos\theta)^{-\frac{1}{2}} \times \cancel{2}\sin\theta$

$= \frac{a\sin\theta}{\sqrt{2-2\cos\theta}}$ which is required [1]

(iii) $\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$

$= \frac{a\sin\theta}{\sqrt{2-2\cos\theta}} \times 2$ when $\theta = \frac{\pi}{3}$

$= \frac{2a\sin\frac{\pi}{3}}{\sqrt{2-2\cos\frac{\pi}{3}}}$

$= \frac{2a\frac{\sqrt{3}}{2}}{\sqrt{1}}$

$= \sqrt{3}a$ cm/min [2]

d) $V = \pi\int x^2\,dy$

$= \frac{\pi}{9}\int_0^{\sqrt{m}}(y^4 - 2my^2 + m^2)\,dy$

$= \frac{\pi}{9}\left[\frac{y^5}{5} - 2m\frac{y^3}{3} + m^2y\right]_0^{\sqrt{m}}$

$= \frac{\pi}{9}\left[\frac{(\sqrt{m})^5}{5} - \frac{2m}{3}(\sqrt{m})^3 + m^2\sqrt{m}\right]$

$= \frac{\pi}{9}\left[\frac{m^2\sqrt{m}}{5} - \frac{2}{3}m^2\sqrt{m} + m^2\sqrt{m}\right]$

$= \frac{\pi}{9} \times m^2\sqrt{m}\left(\frac{1}{5} - \frac{2}{3} + 1\right)$

$= m^2\sqrt{m}\,\frac{\pi}{9} \times \frac{8}{15}$

$= \frac{8}{135}m^2\sqrt{m}\,\pi$

$y = \sqrt{m-3x}$ $\quad x = 0$ $y = \sqrt{m}$

$y^2 = m - 3x$

$3x = m - y^2$

$x = \frac{m-y^2}{3}$

$x^2 = \frac{(m-y^2)^2}{9}$

$= \frac{m^2 - 2my^2 + y^4}{9}$

$\frac{8}{135} \cdot m^{5/2}\,\pi = \frac{5000\pi}{27}$

$m^{5/2} = 3125$

$m = 25$ [4]